Roulette Physics : Basic Insights and Sensitivity Analysis

©J.-I. Eichberger
rouletteball@gmx.de

January 3, 2004

Abstract

This is a companion paper to the article 'Roulette Physics' in which the motion of a roulette ball on a tilted roulette wheel is studied. We give additional background information on the physics of roulette prediction.

KEYWORDS: ROULETTE PHYSICS, PHYSICAL PREDICTION, NEWTONIAN MECHANICS

1 Introduction

The attempt to beat the game of roulette with the help of a computer and an appropriate physical model describing the behaviour of the spinning ball and rotor is not new and there are numerous accounts of this to be found in the literature.

But the physics of roulette are truly fascinating on their own account in that they recapitulate neatly the theory of the damped pendulum and summarize the methods of how to model friction forces and how to treat the problem of a rolling ball on a smooth surface.

REMARK:
In our opinion an appropriate physical model offers advantages over a purely statistical model (e.g. neural networks) as the 'learning' of the critical parameters of a specific wheel can be achieved in much less time.

It is assumed that the reader is familiar with the companion article 'Roulette Physics' which covers the mechanics of roulette prediction through physical methods.

Here we give additional information on the physics of roulette prediction and on the statistical and psychological aspects of achieving a favorable (winning) situation.

2 Basic Insights

2.1 The Statistical Model

When a roulette ball is launched by the croupier it first spins around the rim of the wheel until gravity pulls it down from its track and it spirals into the spinning rotor, usually colliding with one of the vertical or horizontal vanes along the way. This last part of the journey, when the ball is scattered because of the deflectors and then bounces on the rotor before finally coming to rest in one of the numbered pockets makes the game of roulette appear totally random. However, if we take a closer look at the

$$skip = scatter \ + \ bounce$$

behaviour of the ball we find that on most wheels there is some kind of structure. The graph in figure 1 shows a typical skip-statistic obtained from 50 spins on our roulette wheel in the following way: for each spin we recorded the number N1 on the rotor which was under the ball at the moment the ball hit one of the deflectors. Then we noted the winning number N2 when the ball had come to rest in one of the pockets and calculated the skip distance as the number of pockets between the numbers N2 and N1. As can be seen on the above graph, there is a cluster between the skip distances of 8 and 20 pockets.

Approximating the above distribution by a normal distribution we can calculate the parameters of the distribution:

$$mean\ value \ = 16.85\ pockets$$
$$standard\ error = \ 8.75\ pockets$$

Now, if we can predict the position of the rotor head (the green zero) with a standard error of 1.5 pockets and the position of the ball with a standard error of 6 pockets (to be confirmed by experiment), then the total error of the prediction is given by:

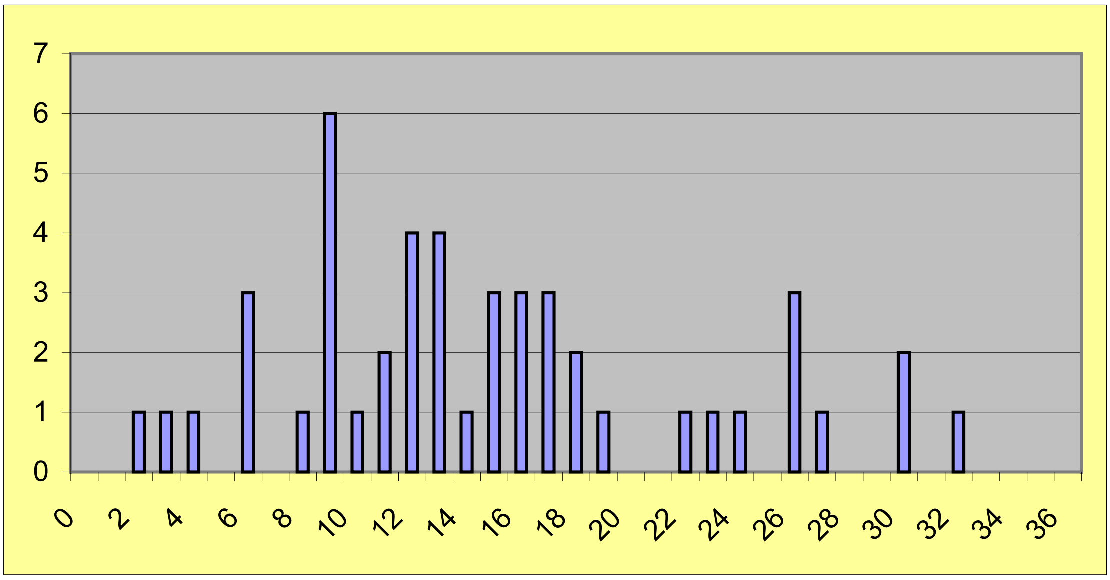

Figure 1: *SKIP - Statistics*

$$total\ error = \sqrt{8.75^2 + 2.25 + 36} = 10.715\ pockets$$

This means that in approximately 67% of the cases the prediction error will be less than ±11 pockets[1].

If we bet on each number of a 5-number sector around the predicted number, then the probability of success will be about 18% according to a z-score of 0.227 (=2.5/11).

For European casinos (where the bet of the winning number goes into the "tronc", i.e. is a compulsory tip for the croupiers), the expectation, i.e. the average rate of return corresponding to a 18% probability of success, is **26**% (c.f. ref.[1] for a more detailed treatment of the topic).

If, under a more demanding assumption, we can only predict in 75% of the cases the number which is under the ball at the moment of collision (and assume a lost cause for the other 25% of the cases), then our expected rate of return would only be **15**% corresponding to a probability of success of 16.5%. The calculation is as follows:

- the standard error for skip and rotor prediction alone is

$$standard\ error = \sqrt{8.75^2 + 2.25} = 8.9\ pockets$$

- a z-score of 0.278 (=2.5/9) gives a probability of success of 22%.

- a probability of success of 0.75·0.22=0.165 gives an expected rate of return of

$$(rate\ of\ return = A \cdot p - 1)$$
$$rate\ of\ return = 7 \cdot 0.165 - 1 = 0.155$$

where p is the probability of success and A (35 times 0.2 units) is the payout of the game (c.f. ref.[1]).

[1] As the individual errors are mutually independant, it is reasonable to assume (approximately) a normal distribution.

2.2 Margin of Safety

While the preceding discussion focussed on statistical concepts which are valid on average when looking at large data sets (i.e. a large number of spins), it must be clear to the individual player that the statistical averages will probably not be realized for a limited number of spins (i.e. during one evening of casino play).

In fact, **the actual rate of return will fluctuate** and might be well below its expected value. The individual player should be aware of these possibilities and should be prepared for them both financially and psychologically.

This realization leads us to the concept of MARGIN OF SAFETY as the central concept for successfull roulette prediction [2].

It is not sufficient to have only a slight edge over the casino; you the player must allow yourself a comfortable MARGIN OF SAFETY.

In the following section we will analyze those situations in which a MARGIN OF SAFETY of **15% to 26% or higher** can be achieved.

REMARK:
For a discussion on intelligent asset allocation for a (favorable) roulette game we recommend reading the chapters pertaining to the Kelley criterion in Dr. Edward Thorps book 'The Mathematics of Gambling' (c.f. ref.[1]).

3 Sensitivity Analysis

3.1 Prediction Method

The method of predicting the winning number comprises three basic steps:

1. predict the location/time when the ball collides with a deflector
2. predict the number of the rotor under the collision deflector at the moment of collision
3. predict the skip distance

The skip = scatter + bounce phase (step 3) cannot be influenced, it can only be measured for a specific wheel. The standard error should be below 9 pockets.

The prediction of the rotor position (step 2) is usually uncritical and can be determined to within a standard error of 1.5 pockets.

However, if the (cumulative) standard error from steps 2 and 3 is significantly above 9 pockets, then we might as well stop our efforts here. Achieving a favorable (winning) situation is then extremely unlikely.

Step 1 involves the prediction of the fall-position, fall-time and the collision point by a physical model and it is for this part of the overall prediction procedure that we will focus on in the following sections.

[2] c.f. ref.[3] for an analogous discussion in relation to security analysis, i.e. being successful in the stock and bond markets

3.2 The Case without Tilt

Here we will look only to the sensitivity in calculating the fall-position θ_f and the fall-time t_f in dependence of the input parameters as this is the most demanding case.

The fall position is calculated by the formula:

$$\theta_f = \frac{1}{2\alpha} \ln \left[\frac{c_1}{\Omega_f^2 - (\beta/\alpha)^2} \right]$$

with

$$x = \frac{e^{\alpha \cdot 2\pi} - \cosh(\sqrt{\alpha\beta} \cdot T_0)}{\sinh(\sqrt{\alpha\beta} \cdot T_0)}$$

$$c_1 = (\beta/\alpha)^2 (x^2 - 1)$$

For parameter values of a typical wheel/ball configuraion of

$$\Omega_f^2 = 7.62$$
$$T_0 = 1.5$$
$$\alpha = 0.0225$$
$$\beta = 0.075$$

the fall-position is $\theta_f = 29.79$ radians (or 4.74 revolutions).

If we program the above formula in a spread sheet application we can vary the individual parameters in order to see how they affect the result.
The most influential parameter by far is the ball deceleration parameter α. Variations of α reflect the 'ball path variatons' which in turn reflect how good an actual roulette wheel fits our physical model.
If, for example, the value for α changes from 0.0225 to 0.230 (0.235) then θ_f changes to 4.61 (4.48) revolutions corresponding to a prediction error of 4.9 (9.5) pockets. An average prediction error of 5 pockets in the fall position will entail an additional error of approximately 2 to 3 pockets in the prediction of the fall time and it is questionable if this can be an acceptable MARGIN OF SAFETY. An average prediction error of 9.5 pockets in the fall-position will produce completely unacceptable results.

CONCLUSION:
On nearly level wheels we can only expect satisfactory results when the manufacturing tolerances of the wheel and ball are very tight resulting in a close match between physical model and reality.

Fortunately, the situation is less critical for...

3.3 The Case with Tilt

The formulas or **roulette equations** for the fall-position and fall-time of the ball which are valid on wheels with or without tilt are given by:

$$c_1 e^{-2a\theta_f} + \eta\left[(1 + \frac{1}{2}(4a^2+1))\cos(\theta_f+\varphi) - 2a\sin(\theta_f+\varphi)\right] + b^2 - \bar{\Omega}_f^2 = 0$$

$$t_f = \frac{1}{ab}\left[c_0 - \text{arsinh}(\sinh c_0 \cdot e^{a\theta_f})\right] - \frac{\eta}{2}\frac{\sin(\theta_f+\varphi) + 2a\cos(\theta_f+\varphi)}{\sqrt{(c_1 e^{-2a\theta_f} + b^2)^3}}$$

For parameter values of a typical wheel/ball configuraion of

$$\begin{aligned}
\bar{\Omega}_f^2 &= 7.62\\
\eta &= 0.193\\
\varphi &= 1.885\\
\alpha &= 0.0225\\
\beta &= 0.075\\
a &= \alpha\\
b^2 &= \beta/\alpha\\
(c_0,\ c_1\ &determined\ from\ T_0\ measurement)
\end{aligned}$$

and different initial conditions T_0, the fall-positions and fall-times are shown in table 1.

By inspecting table 1, several interesting observations can be made:

- The effect of the forbidden zone is clearly visible. For α=0.0225 and T_0=1.55 the ball will come off its track after 4.22 revolutions; for T_0=1.54 the ball will travel 4.74 revolutions. Thus a timing error of 0.01 seconds would in this case result in a prediction error of over half of the wheel. Fortunately, these borderline cases occur rarely in practice.

- Otherwise, small timing errors have little effect on the prediction: if we measure a value of T_0=1.44 instead of T_0=1.45, the total prediction error (fall-position and fall-time) would only be about 1.2 pockets (even for different values of α).

- For T_0=1.44 the corresponding row in table 1 shows the fall-positions and fall-times for different values of α. The different values of α simulate the possible ball path variations for different spins encountered during play. If we use an identified value of α=0.0225 and instead the actual value for the spin is α=0.0230 then we would have a prediction error of app. 3 pockets; if the actual value for the spin is α=0.0235 then we would have a prediction error of app. 6.25 pockets.

CONCLUSION:
On moderately tilted wheels the total prediction error for the fall-position and fall-time will be acceptable, even for relatively large ball path variations (of up to 5%) .

	α=**0.0225**		α=**0.0230**		α= **0.0235**	
T_0	x_f	t_f	x_f	t_f	x_f	t_f
1.30	5.95	10.30	5.88	10.22	5.81	10.13
1.31	5.92	10.31	5.85	10.23	5.77	10.12
1.32	5.90	10.32	5.83	10.23	5.74	10.10
1.33	5.87	10.32	5.80	10.22	5.26	9.07
1.34	5.85	10.33	5.76	10.21	5.18	8.96
1.35	5.82	10.33	5.72	10.18	5.14	8.91
1.36	5.79	10.32	5.20	9.06	5.10	8.89
1.37	5.75	10.30	5.15	9.00	5.07	8.87
1.38	5.23	9.18	5.11	8.97	5.04	8.86
1.39	5.17	9.09	5.08	8.95	5.01	8.86
1.40	5.12	9.05	5.05	8.94	4.99	8.86
1.41	5.09	9.03	5.02	8.93	4.96	8.85
1.42	5.06	9.02	5.00	8.93	4.94	8.85
1.43	5.03	9.01	4.97	8.93	4.92	8.85
1.44	**5.01**	**9.00**	**4.95**	**8.93**	**4.89**	**8.85**
1.45	4.98	9.00	4.92	8.93	4.87	8.85
1.46	4.96	9.00	4.90	8.92	4.84	8.84
1.47	4.93	9.00	4.88	8.92	4.81	8.83
1.48	4.91	9.00	4.85	8.92	4.79	8.82
1.49	4.89	8.99	4.82	8.91	4.75	8.79
1.50	4.86	8.99	4.80	8.89	4.71	8.75
1.51	4.84	8.98	4.76	8.87	4.21	7.67
1.52	4.81	8.97	4.73	8.84	4.16	7.59
1.53	4.78	8.95	4.22	7.72	4.12	7.55
1.54	**4.74**	**8.92**	4.16	7.65	4.09	7.52
1.55	**4.22**	**7.79**	4.12	7.60	4.06	7.50
1.56	4.17	7.70	4.09	7.57	4.04	7.48
1.57	4.13	7.66	4.06	7.55	4.01	7.47
1.58	4.09	7.62	4.04	7.53	3.99	7.46
1.59	4.06	7.60	4.01	7.52	3.97	7.45
1.60	4.04	7.58	3.99	7.50	3.94	7.44
1.61	4.01	7.57	3.97	7.49	3.92	7.43
1.62	3.99	7.55	3.94	7.48	3.90	7.42
1.63	3.97	7.54	3.92	7.47	3.88	7.40
1.64	3.94	7.53	3.90	7.46	3.85	7.39
1.65	3.92	7.52	3.88	7.45	3.83	7.37
1.66	3.90	7.51	3.85	7.44	3.80	7.35
1.67	3.88	7.50	3.83	7.42	3.77	7.32
1.68	3.85	7.49	3.80	7.40	3.74	7.29
1.69	3.83	7.47	3.77	7.37	3.28	6.28

Table 1: tilted wheel: fall-position and fall-time for different values of T_0

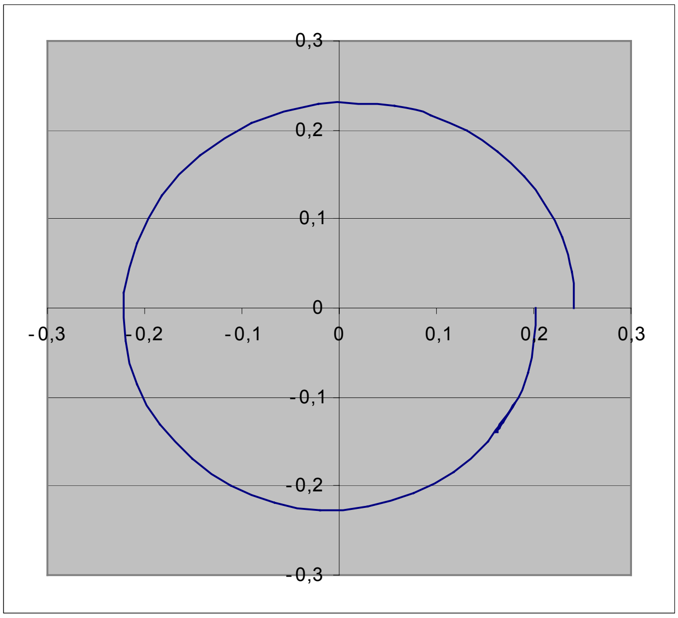

Figure 2: *Rolling Ball in a Tilted Cone*

3.4 Rolling Ball in a Circular Cone

When the ball leaves the rim of the roulette wheel it spirals down towards the rotor until it (usually) hits one of the metal deflectors before hitting the rotor and finally coming to rest in one of the pockets. In low-profile roulette wheels, as they are common in the casinos today, the spiral arc is quite long and its length differs according to the exit location of the ball, the level of tilt and the geometric arrangement of the deflectors.

Table 2 shows a typical collision table for a moderately tilted wheel. Here, the 20 sectors on the rim are numbered 0 to 19 and the 10 ballstops are numbered 0 to 9, with even numbers corresponding to vertical ballstops and odd numbers corresponding to horizontal ball stops.

Because of the low placement of the horizontal ballstops and a stator inclination angle of 14.75 degrees, the ball only collides with the vertical ballstops (for each possible fall-sector).
For the tilt condition at hand there is also a strong bias toward ballstops #0, #2 and #8 (sectors 6 through 14 indicate the forbidden zone).

This bias makes remarkably accurate predictions possible as can be seen from the following heuristic argument.
Suppose that we predict a fall-sector of 3 and a fall-time of 7.66 seconds, a collision ballstop #2 and a corresponding collision time of 2.31 seconds. If instead the ball falls from sector 5 after 7.79 seconds then it will collide with ballstop #2 after 2.11 seconds. This means that the initial prediction error for the fall-position and fall-time of about 6 pockets is nearly compensated (down to about 0.84 pockets) by the corresponding error in calculating the collision time.
So, balls coming off discrete sections (quantums) of the wheel rim will tend to strike the same ballstop at practically the same time and it is this effect that provides for accurate predictions (c.f. ref.[2] for a similar discussion on the 'quantum method' on tilted wheels).

sector	ballstop	time	radius	distance
0	0	2.21	0.2007	1.00
1	0	2.12	0.2041	0.95
2	0	2.01	0.2087	0.90
3	2	2.31	0.1997	1.05
4	2	2.21	0.2017	1.00
5	2	2.11	0.2041	0.95
6	2	2.01	0.2064	0.90
7	2	1.90	0.2085	0.85
8	2	1.79	0.2102	0.80
9	2	1.67	0.2115	0.75
10	2	1.56	0.2121	0.70
11	2	1.45	0.2124	0.65
12	2	1.34	0.2124	0.60
13	2	1.23	0.2123	0.55
14	2	1.12	0.2126	0.50
15	4	1.43	0.2157	0.65
16	8	2.20	0.2038	1.00
17	8	2.10	0.2073	0.95
18	8	2.00	0.2118	0.90
19	0	2.31	0.1980	1.05

Table 2: collision data table

Of course, in some cases the prediction error for the fall-position and fall-time can also be amplified in such a way that the ball collides with a neighbouring ball stop. In our case, because the ball only collides with the vertical ballstops, this would lead to a cumulative prediction error of about 11 pockets ($37/5{=}7.4$ pockets for the position error plus $0.3 \cdot 12{=}3.6$ pockets for the timing error, assuming that the rotor makes 3 revolutions per second).

4 Summary

The guidelines in the section 2 provide a statistical framework for roulette prediction by a physical model such as described in the article 'Roulette Physics'.

In section 3 we give analytical methods for identifying favorable (winning) situations.

However, you will only be able to tell if the procedure works for you after you have made your own tests and are ready to place your bets!

References

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