

# **PERCEPTION OF RANDOMNESS AND PREDICTING UNCERTAIN EVENTS\***

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## **Abstract**

Four types of relatively consistent strategies of predicting uncertain binary events have been identified: (1) a strategy insensitive to short-run sequential dependencies involving the prediction of the long-run majority category –thereafter the long-horizon momentum strategy; (2) a strategy insensitive to short-run sequential dependencies involving the prediction of the long-run minority category- thereafter the long-horizon contrarian strategy; (3) a strategy sensitive to short-run sequential dependencies involving the prediction of the short run majority category- thereafter the short-horizon momentum strategy; (4) a strategy sensitive to short-run sequential dependencies involving the prediction of the short-run minority category- thereafter the short-horizon contrarian strategy. This typology was particularly distinct when individuals were aided with a record of the past sequence of events. The short-term momentum strategy was the most commonly used by respondents. When the type of the binary event was changed from arrows up and down to heads and tails of a coin, implicitly suggesting a random series, this manipulation resulted in an increase in the proportion of the respondents using the short-run contrarian strategy.

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## Introduction

### Uncertain events and their prediction

A necessary condition for trading on the stock market is that some investors sell equities at the same time others buy them. This is evidence of how fundamental beliefs are to human actions. Both sellers and buyers want to increase their wealth. They also share more or less the same information. Therefore, the fundamental difference between these two groups of investors must lie in their beliefs. Sellers believe that stock prices will go down, while buyers predict a stock prices increase. An intriguing question is what makes them differ so much in their beliefs. Our research is mainly motivated by this question.

There are many areas of human activity where success and failure are practically unpredictable and essentially random events. Nevertheless, people strive to predict future events, basing their opinions on past sequences. For example, regardless of the substantial research confirming the random nature of stock price movements (Malkiel, 2003), some investors and their advisors, called technical analysts, try to predict future market movements by scrutinizing past stock price charts. They analyze trends and seek to discover more complicated patterns. Gamblers (both roulette players and horse race fans) make notes of the past performance in order to infer future events from it. Both fishermen and wild mushroom collectors carefully choose “fruitful” places to search, basing their judgments on their past performance.

The concept of randomness is known to be vague. In particular, there is a dispute about equating randomness with equiprobability. The present research concerns random series of binary events without the assumption of equiprobability. On the contrary, we consider series of binary events always having majority (more probable) and the minority (less probable) events.

### Perception of randomness

There have been several studies on how people perceive random binary series. These studies show that people’s notions of randomness are biased in such a way that in a sequence of

random events they expect more alternations and fewer long runs than there are in truly random series (Bar-Hillel, Wagenar, 1991). One of the manifestations of this biased perception of randomness is the negative recency effect, or the gambler's fallacy. It refers to the tendency of people to believe in a higher probability of a given kind of the random event after a long run of the opposite type of event (e.g. predicting a roulette ball falling on the black field after a streak of reds). On some other occasions, people surprisingly exhibit the opposite tendency, called positive recency, where they believe that after a long run of a given event its occurrence is more likely in the next trial (e.g. predicting a roulette ball falling on the red field after a streak of reds, suspecting that the roulette wheel is not symmetrical).

The negative recency effect was typically observed in judgement tasks where various types of sequences were presented to subjects who were requested to select the sequence which in their opinion was really produced by a fair coin (Bar-Hillel, Wagenar, 1991). In several studies it was found (Falk, 1975; Wagenaar, 1970) that subjects tended to choose sequences which did not contain longer series of heads or tails. This means that people tended to overestimate the frequency with which outcomes alternate. For instance, a sequence of heads and tails like HTHHTH was commonly regarded as more random than the sequence HHHHTTT. The same tendency may be observed in the common belief that if an unlikely event occurs once, it is less likely to occur again, e.g. "lightning can't strike twice."

The positive recency effect has been demonstrated in a seminal study by Gilovich et al. (1985) as the so-called "hot hand" belief. This is a belief that hitting one shot makes a basketball player more likely to hit the next shot than if he/she had missed the previous shot. Contrary to this belief of basketball fans, analyses of shooting records (for nine members of the 1980-81 Philadelphia 76ers) showed that players' hit rates were not higher after making their last shots than after missing their last shots.

According to Kahneman and Tversky (1972), a major source of misconceptions regarding randomness is the representativeness heuristic. *"A person who follows this heuristic evaluates the probability of an uncertain event, or a sample, by the degree to which it is (a) similar in essential properties to its parent population and (b) reflects the salient features of the process by which it is generated"* (p. 431). One manifestation of this heuristic is a psychological inclination called "the law of small numbers", according to which even a small sample should be representative of the population from which the sample was drawn. This local

representativeness heuristic allows us to explain the negative recency effect. When an individual believes that a given sequence of events is random, then in accordance with local representativeness, he/she expects that after a series of red in roulette, more black should appear in order to balance both events. The same local representativeness heuristic can also explain the positive recency effect. After a series of events of one type an individual starts to believe that the given sequence of events is nonrandom, and he/she may conclude that this “generator” should produce even more events of the same type.

This suggests that one’s opinion about a source of uncertain events determines which of the two recency effects (negative or positive) applies. When the source generating uncertain events is perceived to be random, then the negative recency effect should prevail. When on the other hand the source is perceived to be non-random, then the positive recency effect should prevail. However, as various observations demonstrate, even in environments commonly regarded as random there are people who try to discover sequential dependencies and to foresee “more probable” events. Some individuals tend to believe that most events they meet in the world have a random nature, while others believe that these same events are of a non-random nature. One purpose of this research was to get to know which of these beliefs prevails, presuming that the source of events remains completely unknown.

## Strategies of predicting uncertain events

Since the middle of the twentieth century scientists and various kinds of analysts have tried to describe the strategies used by people to predict uncertain events. One approach is known as the probability learning paradigm. A typical experiment consisted of a series of randomly generated binary events presented to a subject whose task was to predict each subsequent event after receiving the result of his/her previous prediction. The main finding of this research was that while making a series of predictions, participants tended to match their prediction frequency to the observed frequency of the sequences (the probability matching phenomenon). Probability matching suggests that the subjects learn the underlying probabilities. If those probabilities are correctly inferred, however, then the optimal strategy that maximizes the number of correct guesses is to choose the majority category all the time instead of trying to predict anything.

Various models have been proposed to explain the phenomenon of probability matching, especially as it had been shown that subjects believed in the existence of some sort of

regularity, or pattern, in the sequence of outcomes. In particular, it appeared that individuals tended to show either negative or positive recency effects in their predictions. Restle (1961) discusses some of the typical attitudes of subjects suggesting that “the subject seems to think that he is responding to patterns. Such attempts are natural. The subject has no way of knowing that the events occur randomly, and even if he is told that the sequence is random he does not understand this information clearly, nor is there any strong reason for him to believe it.” (Restle, 1961, p. 109). Generally, research into probability learning does not supply a clear explanation of the probability matching phenomenon. The positive and negative recency effects have not been explained either.

In turn, researchers of the behavior of investors on financial markets distinguish two opposite investment strategies: momentum and contrarian. Momentum investors seek out stocks recently rising in price for purchase or falling in price for sale. Contrarians invest against market trends and do not follow the prevailing consensus view. The definitions of momentum or contrarian investment styles are far from precise. When operationalizing these concepts, Morrin et al. (2002) simply informed their experimental subjects whether the price of a stock in the investor’s portfolio had risen or had fallen in value. However, it is clear that recent rising or falling prices could refer to either shorter or longer -term trends.

The present approach suggests a possible explanation of the prediction process based upon the presumption that an individual keeps records either of local or global frequencies of each binary event, and based on these records he/she forms a subjective probability of the occurrence of the next event. In other words, the subjective probability of the occurrence of the next event can be based either on the short-run of events or on the long-run of events. Consistently, those who record local frequencies should be sensitive to changes of short-run sequential dependencies, and those who record global frequencies should be sensitive to globally observed frequencies of the events.

Furthermore, both those who are sensitive to short-run and long-run sequential dependencies can be divided into two groups. They either tend to predict the events opposite to ones from their history or they tend to predict the events in accordance with their history, presumably believing that the observed sequence is nonrandom and in consequence allows for longer runs of the same event.

In accordance with this we formulate the following hypothesis:

**Hypothesis 1.** *In predicting uncertain events one can identify four relatively consistent strategies :*

- *a strategy insensitive to short-run sequential dependencies involving the prediction of the long-run majority category –thereafter the long-horizon momentum strategy;*
- *a strategy insensitive to short-run sequential dependencies involving the prediction of the long-run minority category- thereafter the long-horizon contrarian strategy;*
- *a strategy sensitive to short-run sequential dependencies involving the prediction of the short run majority category- thereafter the short-horizon momentum strategy;*
- *a strategy sensitive to short-run sequential dependencies involving the prediction of the short-run minority category- thereafter the short-horizon contrarian strategy.*

In the case where hypothesis 1 is confirmed, the question remains concerning which of the above four strategies is most commonly used when the source generating the events in a sequence is unknown to the respondents.

Hypothesis 1 presumes that the generator of the series of events is unknown. In such a situation, an individual can believe that a given sequence is random or not. One can speculate that an individual can try to form a belief about the nature of the series of events through their background knowledge about the type of event. Event types that are conventionally associated with the randomness (eg. coins) might evoke different strategies of predictions than event types without such connotations. Thus we form two other opposing hypothesis:

**Hypothesis 2a.** *Presuming the rationality of human agents, if a series of events is associated by respondents with a random generator, then the respondents will tend to use a long-horizon momentum strategy (because in random sequences local dependencies do not play an important role).*

**Hypothesis 2b.** *Alternatively based on the past empirical research (Burn, Bryan, 2004), if a series is perceived as random respondents will tend to use the short-horizon contrarian strategy (negative recency effect).*

## **Method**

### **Material**

A randomly generated binary sequence of events was created. The events of the sequence were arrows directed downward or upward. The frequencies of the arrows upwards to downwards were set in the ratio 60:40. Thus, upward arrows formed the **majority**

**category**, and downwards arrows formed the **minority category**. There were three versions of the task: (1) arrows (up and down) without the past record on the screen, (2) arrows with the past record on the screen, and (3) a head and tail of a coin without the past record on the screen. In all cases the sequence of events was as shown in the Figure 1. Each respondent was given only one type of task.

The horizontal axis represents the number of events. The vertical axis represents the relation between the upward and downward arrows. On the chart, an arrow pointing upwards is represented by 1, and an arrow pointing downwards is represented by  $-1$ .

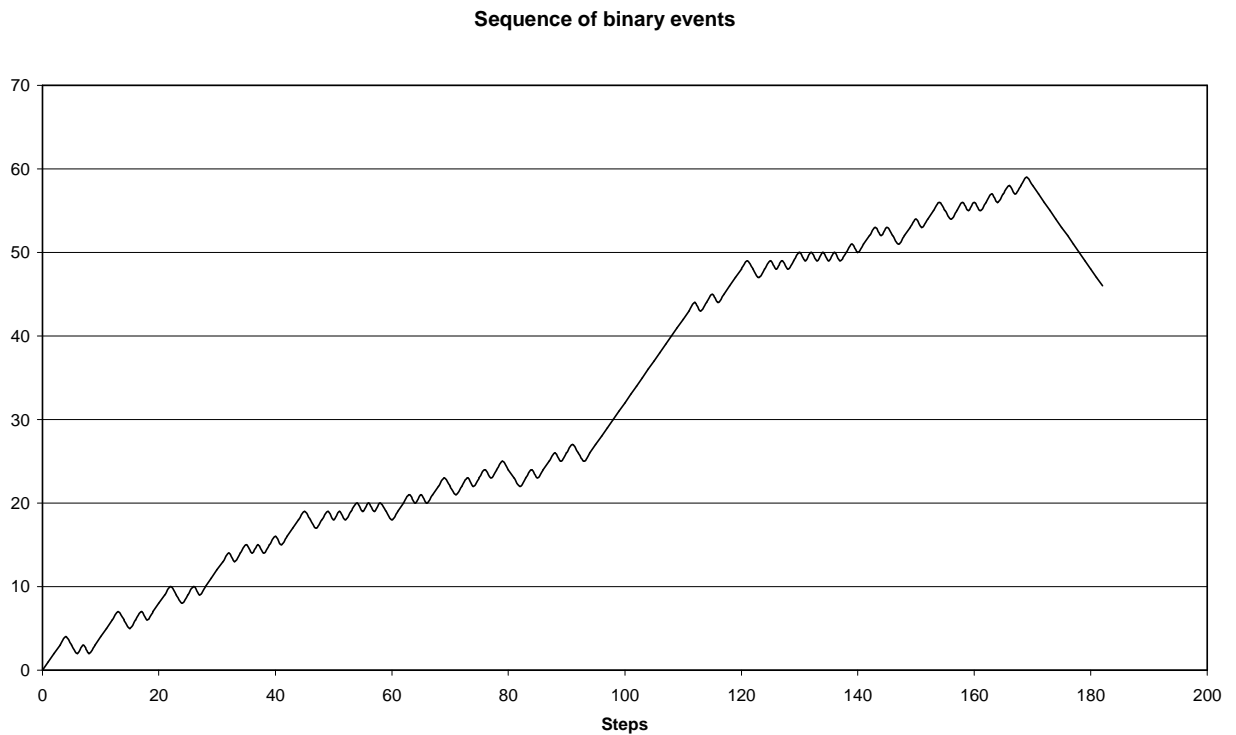


Fig. 1 The sequence of binary events presented to respondents. Plus 1 stands for the majority event (for instance: an arrow up), and minus 1 stands for the minority event (for instance: an arrow down).

In Fig. 1 two smooth (non-random) sections can be seen. The first non-random section consists only of the events of the majority category, while the second section consists only of the events of the minority category.

In the last series of experiments one feature was modified. Instead of arrows, a coin (head or tail) was presented to the subjects.

## Subjects

The respondents were students of two Warsaw universities (a homogenous group). The total number of respondents was 391--113 respondents were presented arrows, 227 respondents were provided with the record of the past sequence, and 151 respondents were presented a sequence of two sides of a coin).

## Procedure

The respondent was asked to observe the events occurring on the screen, and after each 10 of them was asked to make his/her prediction about the next event. The test was created as a kind of a game. Initially each respondent was given 100 points. A proper prediction resulted in one additional point, while a wrong prediction resulted in one point less. A current score was continuously presented on the screen. The test (game) took approximately 5 minutes, as each of 182 symbols was shown on the screen for about 1 second.

## Results

First, we examined two predictions of the respondents within the two non-random sections of the sequence (two vertical lines on Fig.1). Four patterns of predictions are possible in these two points:

- predicting the majority event in both runs,
- predicting the minority event in both runs,
- predicting the majority event in the first run and the minority one in the second run,
- predicting the minority event in the first run and the majority one in the second run.

The first two patterns seem to indicate the insensitivity of a respondent to short-run sequential dependencies and a reaction to the global observed frequencies of the events. We describe them as:

(1) long-horizon momentum strategy – people are insensitive to short-run sequential dependencies involving the prediction of the long-run majority category; and

(2) long-horizon contrarian strategy – people are insensitive to short-run sequential dependencies involving the prediction of the long-run minority category.

The two other patterns of predictions indicate a respondent's reactions to short-run sequential dependencies: (3) predicting the majority event in the first run and the minority one in the second run corresponds to the positive recency effect; and (4) predicting the minority event in the first run and the majority one in the second run corresponds to the negative recency effect.

Then, we tested the reactions of respondents from the four groups to changes in short-run sequences. We classified these sequences into three categories:

- where the proportion of the majority (upward arrows) to the minority category (downward arrows) was almost equal;
- where there was a slight preponderance of the majority over minority category;
- where there was relatively large preponderance of the majority over minority category.

We analyzed prediction tendencies in these three types of sequences for the four groups defined on the basis of their reactions to the two non-random sections. No differences in predictions were found for the two groups which in nonrandom sections reacted to globally observed frequencies of the events. Consistent with their behavior in non-random sequences, the predictions of these groups were completely insensitive to local changes in observed frequencies. Thus, taking together the predictions of these groups in both non-random sequences and in the three types of random sequences, we can state that they have been insensitive to local sequential dependencies.

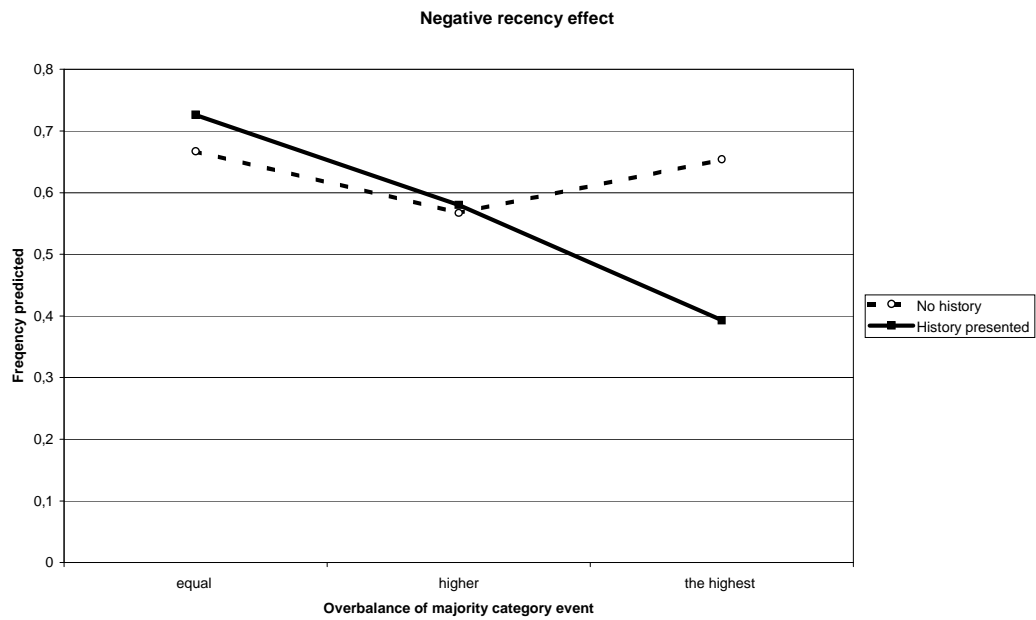


Fig. 2A. Mean percentages of predicting the majority category in three types of random sequences by the respondents who in non-random sections had exhibited the negative recency effect.

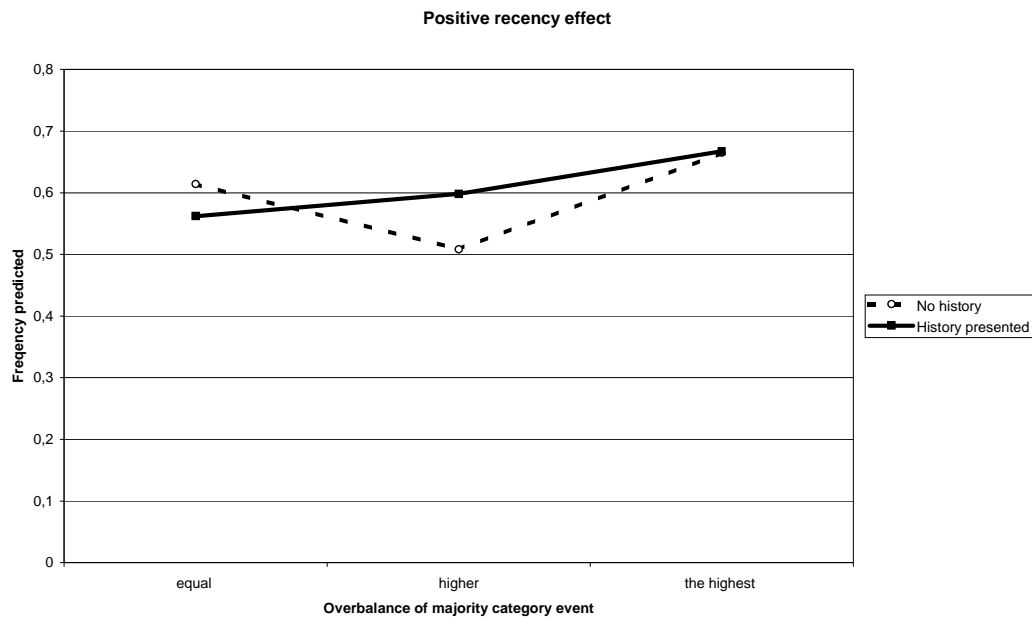


Fig. 2B. Mean percentages of predicting the majority category in three types of random sequences by the respondents who in non-random sections had exhibited the positive recency effect.

Figure 2 shows prediction tendencies in the three types of local sequences made by two groups of respondents: the first which showed the positive recency effect and the second which showed the negative recency effect in the two non-random sections. Let us look at the predictions of the groups which could see the past record on the screen (lines marked as “history presented”). The graphs show that respondents who in non-random sections had exhibited the positive recency effect tended to predict the majority event more and more often as the local section contained more events of majority category. 2 (absence/presence of the past record on the screen x 3 (types of random sequence) ANOVA for percentages of predicting majority category revealed a significant effect of the types of random sequence ( $F=8,64$ ;  $p<0,001$ ) and significant effect of the interaction ( $F=3,33$ ;  $p<0,05$ ). On the other hand, respondents who in non-random sections had exhibited negative recency effect tended to predict the majority event less and less often as the local section contained more events of the majority category. 2 (absence/presence of the past record on the screen x 3 (types of random sequence) ANOVA for percentages of predicting majority category revealed a significant effect of the types of random sequence ( $F=4,64$ ;  $p<0,05$ ) and significant effect of the interaction ( $F=4,40$ ;  $p<0,05$ ). At the same time, the over-all frequencies of predictions in both these groups approximates the probability matching phenomenon. Thus, among those sensitive to local sequential dependencies, we can identify the two relatively consistent strategies of predicting uncertain events:

- Short-horizon momentum strategy – people are sensitive to short run sequential dependencies *resulting in predicting the short run majority category*;
- Short-horizon contrarian strategy - people are sensitive to short-run sequential dependencies *resulting in predicting the short-run minority category*.

An interesting pattern of predictions can be observed in the group which could not see the past record on the screen, i.e. who had to rely on memory and not on perception (red lines). Independent of what effect individuals exhibited in the two non-random sections, we can observe a similar pattern of behavior:

- when the preponderance of the majority category in a local sequence is very small, it is reflected in a higher frequency of predictions of this category,
- when the preponderance of the majority category is a little bit larger, it results in a diminishing frequency of predictions of this category (as if respondents tried to balance both events),

- finally, when the preponderance of the majority category is still larger, it results in an increase in the frequency of predictions of this category (as if respondents tried to adapt their behavior to stimulus frequency).

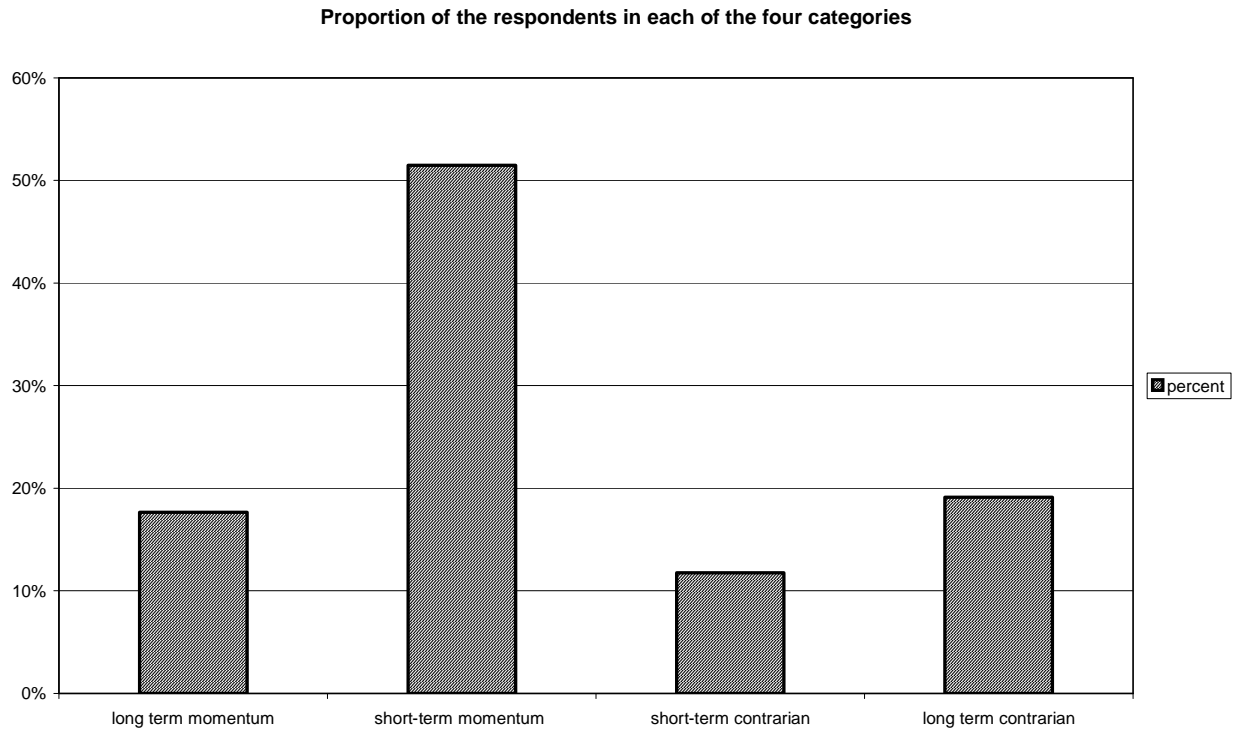


Fig.3 Proportions of the respondents using each of the four strategies of predictions.

Figure 4 shows the proportion of the respondents in each of the four categories. As can be seen, the number of respondents displaying the positive recency effect was larger than for the other groups.

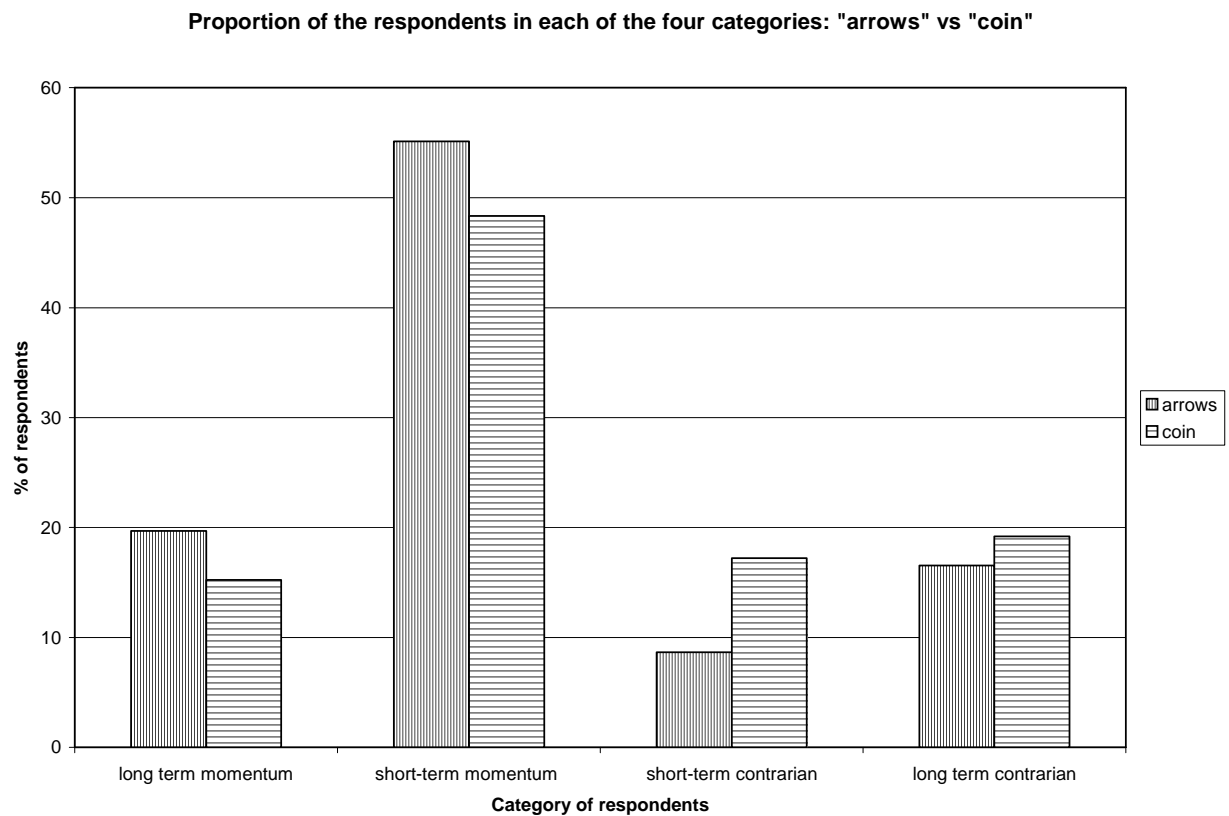


Fig.4 Proportions of the respondents using each of four strategies of predictions, depending upon the perceived source of uncertain events.

Finally, we tested the hypothesis of whether information about the source of uncertain events changes the proportion of people following the trend and going against the trend. As Fig. 4 shows, when a source of uncertain events implies a purely random nature of a series of events (coins), the negative recency effect (the strategy of predicting events opposite to ones forming the recent trend) occurs more frequently than in the case where the nature of the events remains completely unknown. This supports hypothesis 2b and not hypothesis 2a.

## Discussion

### Four types of predicting strategies.

As research on probability learning shows (Restle, 1961), people do not follow a purely rational strategy of predicting the most frequent event all the time. Presumably, they consider predicting the uncertain events as a challenging task and try to experiment with their own skills. Our experiment gives an idea about the strategies people use when performing this challenging task, and more exactly, how people react to short-run sequential dependencies.

The results of our research show that the respondents can be divided into four groups according to two criteria: (1) the length of the horizon taken into account, and (2) the momentum vs. contrarian approach. Although, the four strategies are a kind of idealization, our research shows that the division can be consistently used for the analysis of predictions of uncertain events. Some respondents ignored local trends, whereas others concentrated on them. In both groups, momentum and contrarian strategies were represented.

This suggests that in financial markets, for example, four and not two different investing strategies can be identified. There are investors who display a buy-and-hold strategy, which is the equivalent of our long-term momentum strategy. The long-term contrarian strategy is probably the least common in financial markets, but we can imagine investors who predict a long-horizon trend reversal. The short-term momentum or short-term contrarian strategies are followed by those who use technical analysis as a tool for making predictions. Most technical analysis signals represent the short-term momentum approach, although there are also contrarians who believe that every trend will be sooner or later reversed. As found by Morrin et al. (2002), only around 30% of the analysts examined used the contrarian strategy with a significant majority of around 70% exhibiting a momentum strategy.

In our experiment where the source generating events was unknown, only a relatively small number of the subjects used long-run prediction strategies. A majority of respondents sought local dependencies and tried to make predictions based on these observations. It seems there is a need to seek short-run trends. For example, it is not uncommon for people to form convictions concerning climate changes based only on two or three years worth of observations. Likewise, a “new economy” was declared after only a few years of the bull market at the end of the 20<sup>th</sup> century.

As with the results obtained by Morrin et al. (2002), quite a large majority of our subjects who were sensitive to local dependences followed a short term-momentum strategy, and only a minority followed the short term-contrarian strategy. This observation is in agreement with results of other research. (See for example, Bereby-Meyer, 1997).

### Optimal strategies of predicting the binary events

A crucial question is who uses each of the four strategies described by us above and when do they use them. In order to answer the question we will start with important normative considerations.

Kareev (1995), who explored the optimal strategies of predicting binary events, showed that a rational prediction should depend on two parameters of the sequence: the frequency of binary events and the contingency coefficient which describes the autocorrelation between subsequent events. Since in the case of binary events the contingency measure is identical to the traditional correlation coefficient (see Kareev 1995, p. 492, footnote 1, and Hays 1963, pp.604-606), we will use this latter term.

Our implementation of Kareev's (1995) general analysis is reviewed in the Appendix. The results are summarized in Figure K, where the horizontal axis is the probability of the more frequent event  $\langle 0.5;1 \rangle$ , and the vertical axis is the correlation coefficient  $\langle -1;1 \rangle$ .

Curves 1 and 4 show the maximum values of the correlation coefficient versus the frequency of the more frequent event. Thus, curve 1 shows the boundary of the negative values of the correlation coefficient for particular frequencies. For instance, for a frequency of 0.7, the maximum negative correlation is  $\phi = -0.43$ . Similarly, curve 4 shows the boundary of the positive values of the correlation coefficient versus frequency. One can see an important asymmetry: only for frequencies of 0.5 do negative and positive correlations achieve their maximum possible values of -1 and 1 respectively. As the frequency increases, the maximum negative correlation increases to 0, although the maximum positive correlation remains equal to 1. Thus, if the frequency is higher than 0.5, the positive correlation represents greater potential than the negative correlation.

Curves 2 and 3 represent values of the correlation coefficient beyond which it is profitable to switch from the strategy of always predicting the more frequent event to the strategy of differential use of the most recent event. Curve 2, starting from 0 at a frequency of 0.5, goes

down until it cuts off curve 1 at a frequency of 0.66. Only in the zone between the curves 1 and 2 does the strategy of alternating the most recent event (negative recency effect) dominate the strategy of always predicting the more frequent event. For a frequency higher than 0.66, no negative correlation, not even the strongest possible, is worth noting, as it cannot improve prediction beyond the heuristic of always predicting the more frequent event. In turn, curve 3 covers all frequencies, rising from 0 at a frequency of 0.5 to 0.5 at a frequency of 1. In the zone above this curve, the strategy of predicting the most recent event to repeat itself dominates the strategy of always predicting the more frequent event. This zone is dramatically larger than the zone for predicting the opposite of the most recent event. Moreover, as can be seen, curve 2 has a larger slope than curve 3, which means that positive correlation becomes more useful than negative correlation for making predictions based on correlation.

We can therefore conclude that positive and negative correlations are not equally valuable for making predictions, positive correlations being more useful than negative correlations. As Kareev states: “this is not to say that negative correlations do not exist in reality, but it means that they are less likely to be worth noticing than are positive correlations.” (Kareev, 1995, p 500) Thus, when we do not know what generates events, as in our experiment where neither the frequency of the two events nor the contingency coefficient was known, it is rational for the subjects to exhibit a positive rather than a negative recency effect. Our respondents as a whole behaved in perfect agreement with this conclusion.

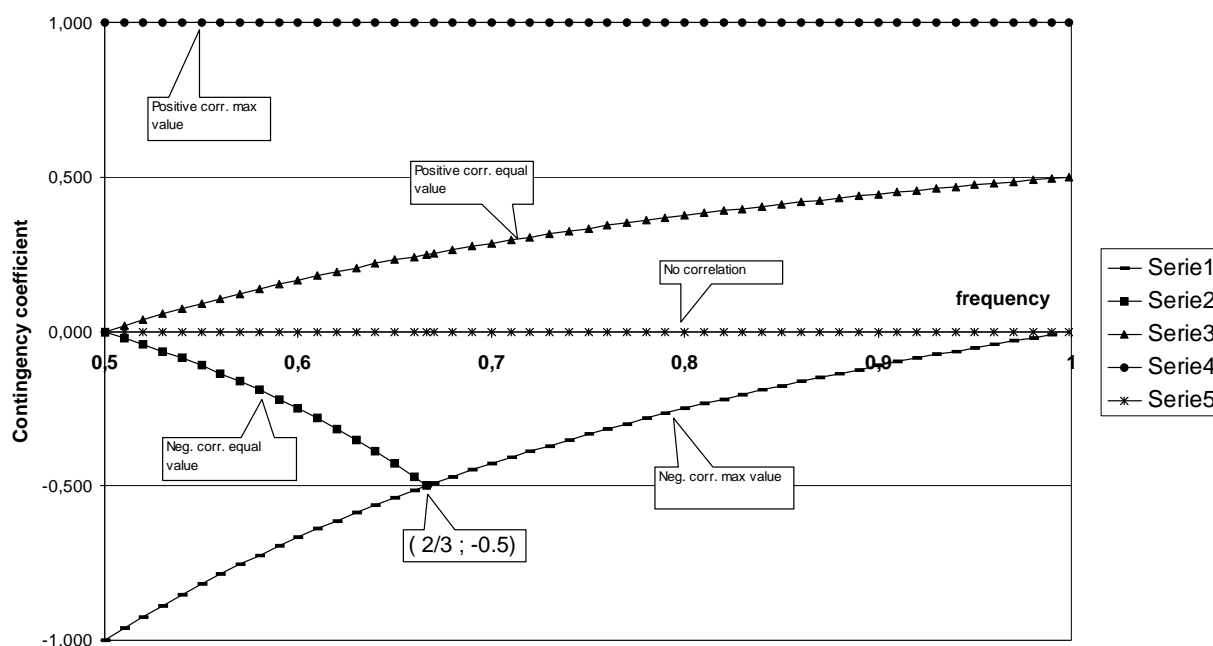


Fig. 5 Optimal rules for predictions of uncertain events by frequency of binary events and level of autocorrelation.

## Who uses short run vs long-run predicting strategies and when?

Taking into consideration the results of our research, we can make a few assumptions concerning this question.

Let's start with a normative statement that for independent events and a stationary process, seeking local dependencies is absolutely irrational. Still, we should expect that people will tend to use the long-run strategies, because in this situation the only reasonable predicting rule is to pay attention to the global frequencies of the events. The optimum strategy is to always predict the more frequent event.

On the other hand, when a process is non-stationary or the binary events are serially correlated, people should use short-run strategies.

Probably there are both people who generally believe in the stationarity of various processes (social and natural) and people who view the environment as changeable.

The first say *nihil novi sub sole* or *historia magistra vitae est*. In our opinion they should express an inclination towards long-run strategies. The others say *panta rhei*, and they should be inclined towards short-run strategies.

Moreover, a long-term strategy is more likely among those familiar with statistics and with random sequences. Therefore, one of the major determinants of using long-run strategies in a random series of independent events is knowledge of the general principles of statistics and personal experience with random sequences. Mathematicians, statisticians, actuaries, meteorologists etc. are probably more immune to analyzing local dependencies than people lacking statistical knowledge and not having any experience with random sequences. On the other hand, those dealing with a world in which probabilistic thinking is rarely observed (e.g. financial analysts) will be keen to look for local regularities in series of events. In fact, in our other research (Tyszka, Zielonka, 2002), we found that in accounting for inaccurate judgments, weather forecasters attach more importance to the probabilistic nature of the events predicted than financial analysts.

Summing up, we think that people who implement long-run strategies are:

- (1) presuming that the environment responsible for the generation of events is stable, and the process itself is stationary with independent events.
- (2) are educated in statistics and exhibit a probabilistic approach to reality (actuaries, weather forecasters)

People implement long-run strategies when:

- (3) they presume that the environment responsible for the generation of events is changing and the process itself is non-stationary or events are strongly sequentially correlated.
- (4) they are not educated in statistics.

## Who uses momentum vs. contrarian strategies and when?

As far as the short-term strategies are concerned, based on the above analysis we may suggest that people should follow a contrarian strategy (and display a negative recency effect) when they believe that events are negatively correlated, and they should follow a momentum strategy (and display a positive recency effect) when they believe that events are positively correlated. The presumptions about those correlations can come either from an assumed mechanism generating the uncertain events or from the record of previously observed sequences of events. For instance, on financial markets the beginning of a long rise of prices may be perceived by investors as the start of a bull market, and they predict a trend

continuation. Some technical analysts maintain that certain formations on price vs. time charts (e.g. so called *head and shoulders*) are predictors of a trend reversal

The question is why, in the very same situation, some people tend to think a streak will continue while other believe it will stop. Indeed, even in situations when the source of uncertainty strongly suggests that observed events should be seen as independent, (as in some conditions of the experiment by Burns and Bryan, 2004) some individuals exhibit a negative recency effect and others a positive recency effect. This suggests that people can differ in their perceptions of the natural and social world. Some people believe that certain events are positively correlated, and the occurrence of a given event means for them that a streak will continue. A good illustration of such a belief is the old Wall Street saying: “Do not fight the tape”.

On the other hand, other people seem to believe that within sequences of events certain events are negatively correlated. Such a belief concerning good fortune in human life is perfectly described in The History Of Herodotus, Volume 1 (Project Gutenberg E-text)

*"Amasis to Polycrates thus saith:--It is a pleasant thing indeed to hear that one who is a friend and guest is faring well; yet to me thy great good fortune is not pleasing, since I know that the Divinity is jealous; and I think that I desire, both for myself and for those about whom I have care, that in some of our affairs we should be prosperous and in others should fail, and thus go through life alternately faring[32] well and ill, rather than that we should be prosperous in all things: for never yet did I hear tell of any one who was prosperous in all things and did not come to an utterly[33] evil end at the last. Now therefore do thou follow my counsel and act as I shall say with respect to thy prosperous fortunes. Take thought and consider, and that which thou findest to be the most valued by thee, and for the loss of which thou wilt most be vexed in thy soul, that take and cast away in such a manner that it shall never again come to the sight of men; and if in future from that time forward good fortune does not befall thee in alternation with calamities,[34] apply remedies in the manner by me suggested."*

According to this description, the mechanism generating human fortunate and adverse life events works as a kind of draw without replacement. Thus, too many fortunate life events increase the probability of an adverse event.

Summing up, we think that people tend to choose the momentum strategy when they believe that the events are not in a random sequence and thus can be somehow positively correlated (e.g. the hot hand effect). People who choose the contrarian strategy believe that the process is non-stationary or events are negatively correlated .

As we mentioned in the introduction, people can show a biased notion of the randomness and expect too many reversals in a sequence of random events. This can result in a tendency to

follow the short-horizon contrarian strategy even in a situation when a sequence is thought to be random. The present research tested the hypothesis that a strategy of predicting uncertain events should be dependent on the interpretation of the event generator. The first group of respondents of our research did not receive any information about the source of the events. The down or up arrows which appeared sequentially on the screen did not give any suggestion to participants about the character of the process, whether it was random or not.

The participants could guess some dependencies by observing the sequence of events.

The second group of respondents faced a kind of manipulation which could influence their interpretation of the character of the process. Instead of the sequence of arrows (up and down) we introduced a sequence of coins (heads and tails). We presumed that a coin would be strongly associated with the concept of randomness. We intended to check the proportion of respondents using each of the four strategies while facing the sequences of arrows and coins.

It turned out that our manipulation resulted in a substantial growth in the proportion of respondents using a short-run contrarian strategy (Kruskal-Wallis ANOVA,  $\chi^2 = 4,044$ ,  $df = 1$ ,  $p < 0,05$ ). This supports our hypothesis 2b. Similar results were obtained in experiments by Boynton (2003) and by Burns and Bryan (2004), in which instructions that the source generated random output increased the likelihood of predicting an alternation in the series. The results show that although perceiving a sequence as random, individuals did not exhibit a rational approach by switching into long-run strategies with no attention paid to local dependencies. Instead, they manifested a stronger inclination toward the negative recency effect.

This is also a confirmation of other research which shows that people's notion of randomness is biased in such a way that in human perception random events are not truly independent, and long streaks are less probable than they are in truly random series (Bar-Hillel, Wagenaar, 1991).

### The impact of the awareness of previous outcomes

Something which deserves our attention is the observation that the four strategies of predicting uncertain events described above were revealed distinctly when the record of the past sequence was presented to the respondents on the screen. Their memory was substantially supported and local dependencies were easier to notice. The respondents from

the group which observed the past sequence of events revealed a consistent attachment to a chosen strategy: momentum or contrarian. (Fig. 2, line: “history presented”)

On the other hand, when the record of the past sequence was not presented to the participants, then all the short-run strategists behaved in the same way (Fig. 2, line: “no history”) When the local frequency of the majority events rose slightly (a point “higher” on the horizontal axis) they all tended to express the contrarian approach. But when the local frequency rose strongly (the point labeled “highest” on the horizontal axis), they all switched to the momentum strategy. We conclude that without observing the past record of the sequence, respondents sensitive to local dependencies behaved as if they initially presumed that the sequence was random. When the frequency of the majority event rose strongly enough they abandoned this view and started predicting the “trend” continuation.

This may be a method for predicting uncertain events that is common to all people. When the past sequence is easily accessible, people choose one of the two strategies, momentum or contrarian, and consistently stick with it.

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## APPENDIX

This appendix provides an overview of the analysis conducted by Kareev (1995) that results in the graph presented in this paper. Kareev generates a basic table,

	C1	C2	total
P1	a	b	a+d
P2	c	d	c+d
Total	a+c	b+d	1

where P denotes predictor and C denotes criterion. Thus, in a sequence of heads and tails such as HTTHHHTH, the first event (H) is the predictor of the ensuing event, called the criterion, (T) and the second event (T) is a predictor of the ensuing event (also T). The predictor P1 and criterion C1 represent the more frequent event (e.g., H in HTTHHHTH), and P2 and C2 represent the less frequent event (T in HTTHHHTH). The entries in the table are the proportions of cases included in the positive and negative diagonal of a contingency table under five conditions. (Kareev 1995, p. 501).

Let  $\omega$  be the probability of the more frequent event (e.g., H in HTTHHHTH). Below we derive the values of a, b, c, and d for each of the five cases. The derivations are straight forward in three of the cases; the derivations are more complex in the remaining two cases. We begin with the straight forward derivations.

### No Correlation between the Predictors and the Criteria

If there is no correlation between P1 and P2, on the one hand, and C1 and C2, on the other, then we have  $P(C_i/P_j) = P(C_i)$  for all i and j. The values of a, b, c, and d are then as follows:

$$\begin{aligned} a &= P(P1)P(C1/P1) = \omega\omega = \omega^2 \\ b &= P(P1)P(C2/P1) = \omega(1-\omega) \\ c &= P(P2)P(C1/P2) = (1-\omega)\omega \\ d &= P(P2)P(C2/P2) = (1-\omega)(1-\omega) = (1-\omega)^2 \end{aligned}$$

### Positive Correlation of Maximum Value

If there is positive correlation, then  $P(C1/P1) > P(C1)$  and  $P(C2/P2) > P(C2)$ . If the correlation is of maximum value, then  $P(C1/P1) = P(C2/P2) = 1$ , and, thereby,  $P(C1/P2) = P(C2/P1) = 0$ . The values of a, b, c, and d are then as follows:

$$\begin{aligned} a &= P(P1)P(C1/P1) = \omega 1 = \omega \\ b &= P(P1)P(C2/P1) = \omega 0 = 0 \\ c &= P(P2)P(C1/P2) = (1-\omega)0 = 0 \\ d &= P(P2)P(C2/P2) = (1-\omega)1 = (1-\omega) \end{aligned}$$

### Negative Correlation of Maximum Value

If there is negative correlation, then  $P(C1/P2) > P(C1)$  and  $P(C2/P1) > P(C2)$ . If the correlation is of maximum value, then  $P(C1/P2) = 1$ , and, thereby,  $P(C2/P2) = 0$ . Note that we cannot have  $P(C2/P1) = 1$  because  $P1$  is more frequent than  $C2$ , and thereby they cannot be matched in a 1 to 1 fashion. From  $P(C1/P2) = 1$ , we have  $c = P(P2)P(C1/P2) = (1-\omega)1 = (1-\omega)$ , and from  $P(C2/P2) = 0$ , we have  $P(P2)P(C2/P2) = (1-\omega)0 = 0$ . Since  $b+d = P(C2) = (1-\omega)$ , we have  $b = (1-\omega)$ , and since  $a+b = \omega$ , we have  $a = 2\omega-1$ . The values of  $a$ ,  $b$ ,  $c$ , and  $d$  are then as follows:

$$a = P(P1)P(C1/P1) = 2\omega-1$$

$$b = P(P1)P(C2/P1) = 1-\omega$$

$$c = P(P2)P(C1/P2) = 1-\omega$$

$$d = P(P2)P(C2/P2) = 0$$

### Positive Correlation with Equal Values Between the Prediction Rules

If there is positive correlation, then  $P(C1/P1) > P(C1)$  and  $P(C2/P2) > P(C2)$ . If there is equality between the two prediction rules, then  $a+d = P(C1)$ . To see this, note that  $a+d$  is  $P(P1)P(C1/P1) + P(P2)P(C2/P2)$  and is thereby the frequency of correct predictions given the  $p = 1$  prediction rule. Recall that  $P(C1)$  is the frequency of correct predictions under the  $p = 0$  rule. The contingency table remains undetermined, but by summing on rows and columns, the table must have the form

	C1	C2	total
P1	a	$\omega-a$	$a+c = \omega$
P2	$\omega-a$	$1-2\omega+a$	$c+d = 1-\omega$
total	$a+c = \omega$	$b+d = 1-\omega$	1

From  $a+d = P(C1)$ , we have  $a+(1-2\omega+a) = \omega$ , so that  $a = \frac{3\omega-1}{2}$ . The values of  $a$ ,  $b$ ,  $c$ , and  $d$  are, by

substitution, as follows:

$$a = P(P1)P(C1/P1) = \frac{3\omega-1}{2}$$

$$b = P(P1)P(C2/P1) = \frac{1-\omega}{2}$$

$$c = P(P2)P(C1/P2) = \frac{1-\omega}{2}$$

$$d = P(P2)P(C2/P2) = \frac{1-\omega}{2}$$

### Negative Correlation with Equal Values Between the Prediction Rules

If there is negative correlation, then  $P(C1/P2) > P(C1)$  and  $P(C2/P1) > P(C2)$ . If there is equality between the two prediction rules, then  $b+c = P(C1)$ . To see this, note that  $b+c$  is  $P(P1)P(C2/P1) + P(P2)P(C1/P2)$  and is thereby the frequency of correct predictions given the  $\rho = -1$  prediction rule. Recall that  $P(C1)$  is the frequency of correct predictions under the  $\rho = 0$  rule. The contingency table is again undetermined, and again, by summing on rows and columns, the must have the form

	C1	C2	total
P1	a	$\omega - a$	$a + c = \omega$
P2	$\omega - a$	$1 - 2\omega + a$	$c + d = 1 - \omega$
total	$a + c = \omega$	$b + d = 1 - \omega$	1

From  $b+c = P(C1)$ , we have  $(\omega - a) + (\omega - a) = \omega$ , so that  $a = \frac{\omega}{2}$ . The values of a, b, c, and d are, by substitution, as

follows:

$$a = P(P1)P(C1/P1) = \frac{\omega}{2}$$

$$b = P(P1)P(C2/P1) = \frac{\omega}{2}$$

$$c = P(P2)P(C1/P2) = \frac{\omega}{2}$$

$$d = P(P2)P(C2/P2) = \frac{2 - 3\omega}{2}$$

The contingency coefficient, denoted by  $\phi$ , is calculated as follows:

$$\phi = \frac{ad - bc}{\sqrt{(a+b)(c+d)(a+c)(b+d)}}.$$